An alternate approach to study the spatial coherence of the positrons is to measure the two-point correlation function of the image obtained from an intense, short pulse of positrons. A specific model would be to assume that the positrons in the solid form a noninteracting Fermi gas at a temperature T. Owing to the nearly dissipationless negative-affinity positron emission mechanism, and the large size of the negative affinity compared to both the thermal energy kT and the expected positron Fermi energy E_F, the in-focus microscope image of the positrons emitted from the surface over a time T is a direct measure of the positron distribution just beneath the metal surface. Thus the two-point correlation function of the microscope image is approximately the time-averaged 2D projection of the two-point correlation function $G(x)$ of the positrons in the metal. For a completely degenerate noninteracting positron gas, the latter is given by [96]

$$G(x) = \frac{n^2}{\Omega^2}\left[1 - \frac{9}{2}\, j_1^2(xk_F)/x^2\right], \tag{35}$$

where $j_1(z)$ is a spherical Bessel function of the first kind. At $x = 0$, $G(x)$ dips to half the classical value n^2/Ω^2. For completely polarized positrons, the factor of $9/2$ would be replaced by 9, and $G(0) = 0$. At nonzero temperatures, the dip is reduced by a factor of roughly $E_F^{3/2}/(E_F^{3/2} + kT^{3/2})$, and its width is roughly $\Delta x \approx [m(E_F + kT)]^{-1}$. Departures of the measured G_{exp} from the model calculation would inform us about various interesting aspects of how a dense gas of positrons interacts with a real metal surface. Notable would be effects due to screening, inelastic processes, lack of thermalization, surface irregularities, impurity scattering and post-emission Coulomb repulsion amongst the positrons.

(c) Discuss the use of speckle microscopy or micro low-energy lepton diffraction (micro LEPD) for obtaining atomic resolution information about sample surfaces. In regular LEPD the sample surface is coherent over a scale much greater than the transverse coherence of the positron beam. Measurements of the diffracted beam intensities *vs.* beam energy may be compared with theoretical calculations to determine the arrangement of the atoms in the first few atomic layers of the single-crystal sample. If the probing beam is perfectly coherent, the diffracted intensity *vs.* parallel momentum transfer and beam energy for a nonperiodic surface may be treated in the same manner as LEPD to solve for the surface structure.

10) Update the discussion on the thermal desorption of Ps_2 molecules in ref. [72] by including the improved estimate of the Ps_2 binding energy. What are the optimum temperature, surface orientation of an Al sample and positron density for producing Ps_2? Consider how the triplet Ps annihilation signal would change as one adjusts the focus of a pulsed positron microbeam.

Note that Ps_2 annihilates mostly into two photons, whereas the Ps that is thermally desorbed from a surface annihilates mostly into three photons.

11) What are the eigenstates of the positronium molecule Ps_2 in its lowest-energy configurations? By this I mean classify the states according to the quantum numbers of S, P and C for $L = 0$. Here S is the total spin operator, P is the parity operator, C is the charge conjugation operator and L is the total orbital angular momentum. Denoting the states by $|S, M_S, P, C\rangle$ there are five magnetic substates $|2, M_S, +1, +1\rangle$ made up of triplet positronium atoms with $M_S = -2, -1, 0, 1, 2$. For $S = 1$ there are three possibilities: three odd-parity magnetic substates $|1, M_S, -1, +1\rangle$ made of triplet positronium atoms; and two sets of three states that are half singlet and half triplet positronium and thus have odd C-parity, namely the even-parity substates $|1, M_S, +1, -1\rangle$ and the odd-parity substates $|1, M_S, -1, -1\rangle$. Finally there are two states with $S = 0$ having all the same quantum numbers, one made of triplets $|0, 0, +1, +1\rangle_{\text{triplet}} \equiv |t\rangle$ and one made of singlets $|0, 0, +1, +1\rangle_{\text{singlet}} \equiv |s\rangle$. Show that

$$|t\rangle = 3^{-1/2}\{\,|1, +1\rangle|1, -1\rangle + |1, -1\rangle|1, +1\rangle - |1, 0\rangle|1, 0\rangle\} \tag{36}$$

and that

$$|s\rangle = |0, 0\rangle|0, 0\rangle\,. \tag{37}$$

Here the positronium states are designated by the symbols $|J, M\rangle$. Define the sum and difference of the two states

$$|\zeta_\pm\rangle = |s\rangle \pm |t\rangle\,. \tag{38}$$

$|\zeta_+\rangle$ and $|\zeta_-\rangle$ will be bonding and antibonding states for Ps_2, and the ground state will be mostly $|\zeta_+\rangle$ with a very small admixture of $|\zeta_-\rangle$. Here I am assuming that the hyperfine interaction due to the spins of the four particles can be neglected.

So far we have ignored the spatial components of the wave function of the four particles. In the approximation that we have two positronium atoms far apart at a distance R the eigenenergies of the two states $|\zeta_+\rangle$ and $|\zeta_-\rangle$ will be roughly the potential-energy curves for the interaction of two positronium atoms, $V_\pm(R)$. For large R, $V_+ \approx V_-$. In fact, for large R, the eigenstates will be $|s\rangle$ and $|t\rangle$ and their energies will differ by twice the positronium hyperfine interval or about 2 meV. During a collision of two triplet positronium atoms a phase shift between the $|s\rangle$ and $|t\rangle$ states

$$\Delta\phi = \int \{V_+(R(t)) - V_-(R(t))\}\, \mathrm{d}t/\hbar \tag{39}$$

will develop that depends on the relative kinetic energy. After the collision there will be a nonvanishing amplitude for the positronium atoms to be in the singlet state unless the initial state was purely the $S = 2$ state. At low enough relative kinetic energies, the transition from two triplet atoms to two singlet atoms will be suppressed because the energy change of two hyperfine intervals has to be transferred from the spin system to the center of mass of each of the two positronium atoms. Make all these arguments more rigorous, estimate $V_\pm(R)$ and the positronium-positronium scattering cross-sections.

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