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Physics with dense positronium

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Our recent experiment on Ps-Ps spin exchange quenching represents the first time that two Ps atoms have been observed interacting with each other. The implication is that experimentation on various systems containing more than one Ps atom is now possible, including the Ps_2 molecule, the Ps Bose-Einstein condensate, the Ps atom laser and the annihilation gamma ray laser. Here we describe various aspects of such research, including the development of new techniques, work currently underway and some possible future experiments.

1 Introduction The invention of the buffer gas positron trap by Cliff Surko and co-workers in the late eighties [1] led to the exploration of a number of new experimental regimes [2]. In particular, the use of long lived positron plasmas was a key component in the creation of antihydrogen [3]. By taking such a plasma and compressing it in time to a sub ns width, we have been able to generate intense positron bursts with an instantaneous current of ~ 10 mA. These bursts are also compressed spatially using time varying electric and magnetic fields to a radius of ~ 100 microns.

Implanting such an intense pulse into a porous silica material led to the creation of a high density (around 2×10^{15} cm^{-3}) of confined positronium (Ps) atoms, such that they were able to interact with each other [4]. An increase of only 3 orders of magnitude in density should be sufficient for the production of a positronium Bose Einstein condensate (BEC) [5] with a critical temperature of 15 K [6]. The present areal density should be sufficient to produce di-positronium (Ps_2) molecules on the surface of a clean single crystal [7]. A resonant laser excitation of the Ps_2 molecule to the 1S1P state [8] would provide an unambiguous signal.

As well as studying dense positronium effects intense positron pulses may be used for studying transient phenomena. In one experiment, paramagnetic centers were created in a porous silica material and probed using a positron pulse. Since unpaired spins can lead to spin exchange quenching, the decay rate of positronium can be used as a measure of the density of paramagnetic centers, and if such centers are created at a well defined time (for example, by laser irradiation) their lifetimes may be measured.

Here we describe the creation of the intense positron beam and then outline some of the experiments we expect to be able to do with it in coming years.

2 Creation of an intense positron pulse The positron beam is derived from a ^{22}Na source and a solid neon moderator. This beam is used to supply a two stage Surko trap which then supplies a UHV positron accumulator [9] at 4 Hz. A schematic of the apparatus is shown in Fig. 1. Because the accumulator stage is kept at a relatively low pressure (~ 2×10^{-8} Torr of cooling gas, with a base pressure better than 5×10^{-11} Torr) the trapped positrons have a long lifetime (> 1000 sec) and up to 100 million can be accumulated and stored.

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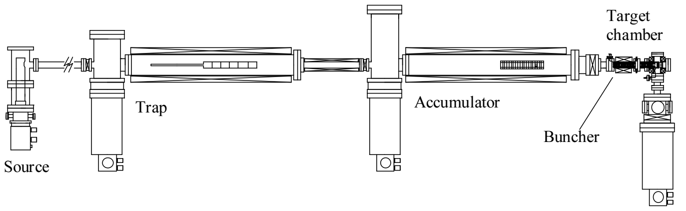

Fig. 1 A schematic of the positron source, trap, accumulator and target chamber.

The electrode structure of the positron accumulator is a multi-ring Penning trap, with an axial magnetic field of around 500 G. The accumulation region is biased as a harmonic potential well during the filling phase, and a radially segmented electrode is used to apply a rotating electric field, which causes plasma compression [10].

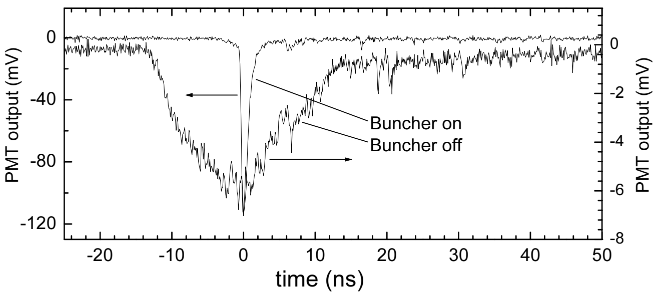

Fig. 2 Positron beam time compression with and without the buncher.

When filling is complete the positions are ejected from the trap by rapidly applying a 200 V pulse along the multi-ring structure. The dump pulse, however, is configured to form only half of a potential well so that the positrons are ejected along a parabolic potential and are thus focussed in time [11]. This leads to a pulse width of around 10-20 ns, depending on the number of positrons in the pulse. If a shorter pulse is required a second buncher may be used just after the accumulator. This is essentially the same process as the initial accumulator pulser, but with a 3 kV pulse the beam width is largely insensitive to the positron density, and a sub ns pulse is created. Figure 2 shows the beam width with the secondary buncher on and off, as measured by a PbF_2 Cherenkov radiator attached to a fast photomultiplier.

The positron plasma is around 1 mm (FWHM) in the positron accumulator and so a pulsed 1 T magnet is used near the sample region to compress this further. This strong field is enough to magnetically quench positronium in the m = 0 substate, but does not affect the m = ± 1 substates, and thus only around 1/3 of the Ps signal is lost due to the magnetic field. Figure 3 shows the beam profile with the pulsed magnet on and off. This was measured by implanting the beam into a phosphor screen and recording the image with a CCD camera.

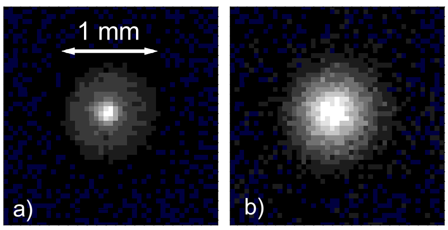

Fig. 3 Beam profiles at high (a) and low (b) density.

3 Single shot measurements Measuring the annihilation radiation from an intense positron burst requires some rather different techniques from those usually employed in positron studies [12]. Using a fast scintillator (or a Cherenkov radiator) with a low light output we are able to measure positron annihilation lifetime spectra in one burst. The signal observed when long lived triplet positronium undergo spin exchange quenching by interacting with each other is simply an apparent reduction in the amount of triplet Ps. Due to the limited detector time resolution (which is ~ 15 ns when a $PbWO_4$ scintillator is used) [13] annihilations that occur within about 10 ns of the positron pulse are indistinguishable and will be contained in the prompt peak.

Similarly, there is no difference between the signal for two positronium atoms decaying quickly following a spin exchanging collision or two Ps atoms forming Ps_2 and then decaying with the spin averaged decay rate. Figure 4 shows the difference in the Ps annihilation lifetime spectra for a high density positron pulse and a low density pulse.

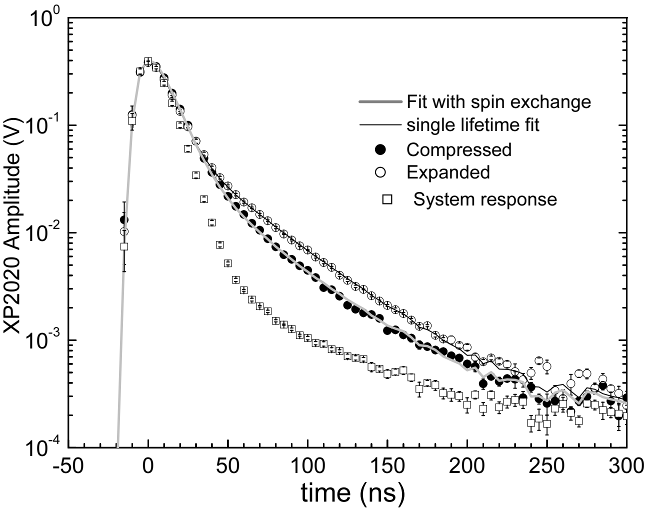

Fig. 4 The reduction in the long lived Ps when the beam is compressed to a high density.

The implied cross section for spin exchanging interactions between Ps atoms is higher than expected, and these data may show both spin exchange quenching and Ps_2 formation. Figure 5 shows the reduction in the delayed fraction (that is, the amount of long lived Ps present) as a function of the areal density. The expected linear dependence is observed. More work is required to provide an unambiguous Ps_2 signal, and it will probably require some sort of surface mechanism.

Shorter lifetimes can also be measured if a PbF_2 Cherenkov radiator is used instead of a scintillator. In this case the signal is diminished because Cherenkov radiation is not efficiently created by 511 keV gamma rays. Furthermore the Ps signal consists mainly of photons with energies below 511 keV and so the sensitivity to positronium is even weaker. Nevertheless, we have positron pulses of sufficient intensity that a useful signal can be obtained using such a detector. Because Cherenkov radiation is emitted almost instantaneously the time response in this case is determined by the photomultiplier tube (PMT) used. This is usually somewhere in the range of 2-5 ns depending on the type and condition of the PMT. Figure 6 shows data obtained using a Teflon sample where a ~ 3 ns lifetime component may be present. A very short time resolution actually reveals the presence of cable imperfections and leads to an irregular looking lifetime spectrum. However, provided that conditions are maintained these apparent irregularities can be included in the resolution function and satisfactory fits obtained.

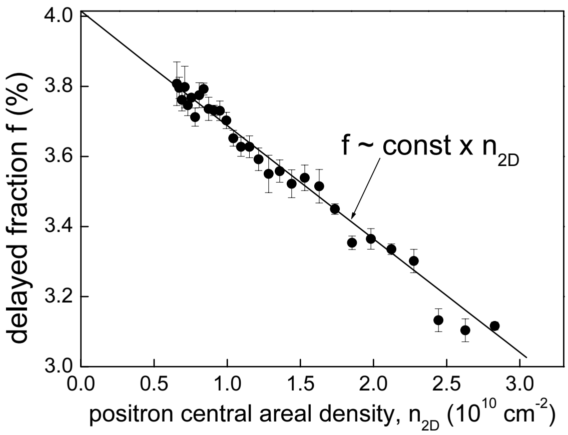

Fig. 5 The reduction in the amount of long lived Ps is a linear function of the areal density.

By introducing a laser system it is also possible to use this method to measure transient phenomena. In this case a high spatial density is not required, but the ability to record an entire lifetime spectrum in a single shot does allow for the measurement of phenomena that have characteristic time scales ranging from ns to days. Using a porous silica sample we performed such an experiment.

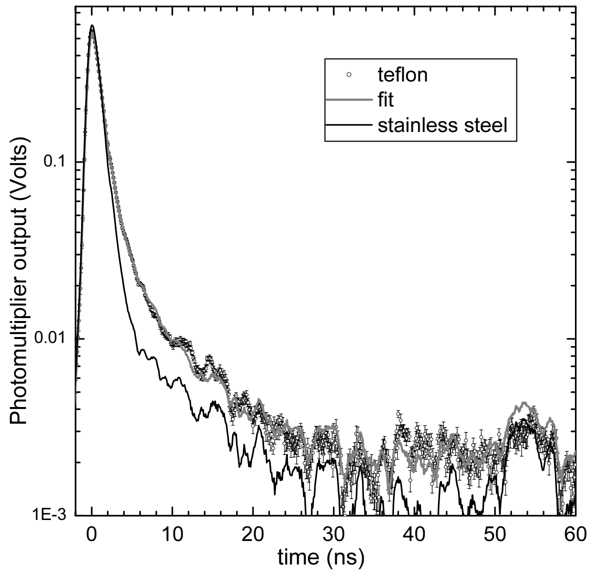

Fig. 6 A single shot lifetime spectrum with Teflon and a stainless steel target, the latter used as a system response function which was then used in the fitting procedure.

The sample was created with attention to the porous structure and was not annealed or otherwise treated to remove, or even characterize contaminants. Thus it is very likely that a host of impurities were present on the internal surfaces of the sample. It is well known that laser irradiation of sample containing impurities can lead to the production of paramagnetic centers of various kinds [14]. Using a relatively low power laser pulse from an Nd YAG laser we measured the delayed fraction as a function of time after laser irradiation. A schematic of the laser timing arrangement is shown in Fig. 7.

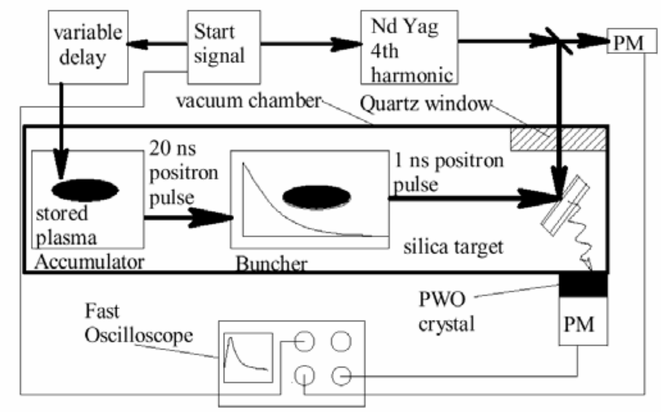

Fig. 7 A schematic of the layout of the laser timing and gamma ray detection system.

The positron pulses themselves did not generate any paramagnetic centers and so for very long delays (> 10 sec) the data were collected by successive positron shots after a single laser irradiation pulse. It was found that the paramagnetic centers created by the laser decayed logarithmically over many orders of magnitude. Figure 8 shows the delayed fraction as a function of time for various laser powers and wavelengths.

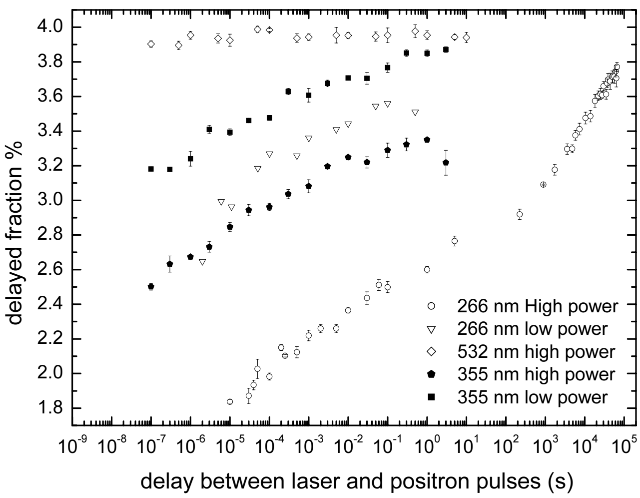

Fig. 8 The delayed fraction as a function of time for various laser powers and wavelengths. Low power refers to ~ 3 mJ/pulse, and high power ~ 10 mJ/pulse.

Because Ps in the silica pores cannot re-enter the bulk this method provides a unique technique for measuring the lifetime of *surface* paramagnetic centers. To our knowledge there is no other way to do this as electron spin resonance or optical spectroscopy methods do not differentiate between bulk and surface paramagnetic centers.

4 Future work One of the main goals of our research is the creation of a Ps BEC. Although this is undoubtedly a challenging experiment, our recent progress has been encouraging and many of the expected obstacles have been overcome. Nevertheless, this section is necessarily speculative in nature.

The essence of the phenomenon of Bose-Einstein condensation for weakly interacting particles is that below a certain critical temperature T_c a macroscopic fraction of a collection of identical particles obeying Bose-Einstein statistics collects into the single lowest energy state, whether it be the zero momentum state in free space or the ground state for a system in a confining potential [15, 16]. Condensation will only occur if the density of particles is higher than a certain threshold at a given temperature. If the density of states increases sufficiently rapidly with energy, the number of particles in the ground state n_0 will exhibit a sudden transition to a finite number in the sense that $\partial n_0 / \partial T = 0$ for $T > T_c$ and $-\partial n_0 / \partial T > 0$ for $T < T_c$ in the thermodynamic limit where there are macroscopic numbers of particles in the system. The dynamical reason [17, 18] for the condensation of weakly interacting Bosons is that the rate for a particle obeying Bose-Einstein statistics to scatter into any state is proportional to one plus the number of particles already in that state. As the temperature gets lower, the lowest energy state is more and more likely to have particles in it and scattering into this state becomes a runaway process below a critical temperature, with the condensate forming after a time equal to the inverse of the collision rate times factors depending on the number of particles and whether "condensing" includes the development of off-diagonal long range order or not. In the case of a gas of polarized m = 1 triplet positronium atoms in a cavity, the positronium-positronium collision rate at a density of 10^{18} cm^{-3} at 10 K will be about 3×10^9 s^{-1} [19], which will ensure that a macroscopic occupation of the ground state occurs in a time short compared to the 142 ns ortho-positronium annihilation lifetime.

If BEC occurs for a gas of free particles in infinite space or in a container with periodic boundary conditions, the lowest energy state is the one with zero momentum. In three dimensions the critical temperature for Bose-Einstein condensation of weakly interacting free particles is given by

$$kT_c = [ng_{\frac{3}{2}}(1)]^{\frac{2}{3}} 2\pi\hbar^2 / m \approx \left[n/10^{18}\,cm^{-3}\right]^{\frac{2}{3}} \times 14.62K\,,$$

where $g_l(z)$ are the Bose-Einstein functions of order l, z is the fugacity, and the explicit evaluation is for a gas of positronium atoms of density n and mass $m = 2m_e$ all in the same state of total spin S and S_z. If the particles have a repulsive interaction, a condensed system much larger than the interparticle momentum scattering mean free path may exhibit superfluid properties because the phonon modes exhaust all the degrees of freedom for low energy excitations and objects traveling at velocities sufficiently less than the speed of sound can emit no Cherenkov phonons [20]. BEC has also been discussed for a variety of other situations, such as a dimensionality other than three, free particles in a confining potential [21] and the "Bose-glass" formed from bosons in a disordered potential such as liquid He^4 in porous Vycor [22, 23].

For many years the only known example of Bose-Einstein condensation was liquid Helium, for which the fraction of particles in the zero momentum state is only about 10% because of the strong interactions of the atoms in the liquid state. It was therefore impossible to study certain aspects of BEC such as the expected critical phenomena near T_c and the effects of a weak impurity potential. The achievement of BEC of atoms in an optical trap [24, 25] has allowed many interesting effects to be studied in a nearly ideal Bose gas, including observation of the interference of two separated condensates [26], emission of atoms from a condensate in a coherent beam (the atom laser) [27], tuning of the particle interactions from repulsive to attractive, and pairing interactions in a gas of atoms obeying Fermi statistics [28]. The achievement of atomic BEC has been accompanied by an explosion in other technological uses of atom traps, including the atomic fountain clock [29], sensitive gravimeters based on atom interferometry [30], quantum computation schemes [31] and unbreakable quantum cryptography methods [32].

Figure 9 shows a 5 micron wide positron beam impinging on a cavity in a silica matrix. If Ps is formed in the typical way this should produce a density around 10^{18} cm^{-3} and the critical temperature would then be around 15 K. The details of the most appropriate material to use are still being investigated, but in light of our results regarding laser induced paramagnetic centers it seems likely that some care will be needed to make sure that the final cavity does not contain excessive impurities, from which paramagnetic centers may arise.

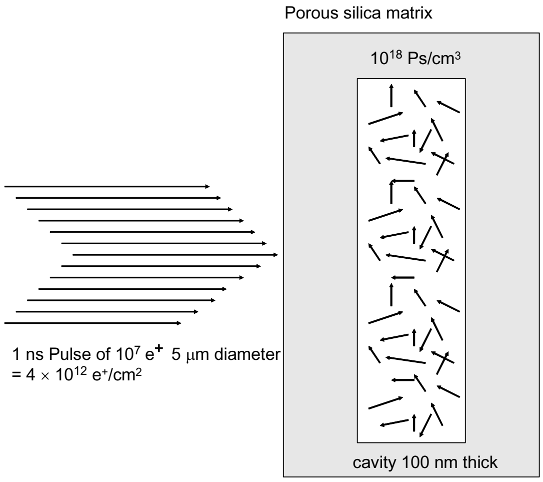

Fig. 9 10 million positrons in a 5 micron pulse impinging on a small cavity should generate Ps at a density around 10^{18} cm^{-3}. The positronium will rapidly thermalize to the ambient (cryogenic) temperature and form the condensate.

The state of positronium in our porous structures may be determined by imaging the absorption of laser light at the positronium Lyman-alpha wavelength. Positronium Lyman-alpha or 1S-2P transitions are induced by laser light at 243.021 nm. For cw excitation exactly on resonance the Rabi frequency is $\Omega_R = \left(\frac{eEd}{2\hbar}\right)$, where E is the amplitude of the laser electric field and $d \equiv \langle 1S|z|2P\rangle = \left(\frac{2}{3}\right)^5 a_0 2^2\sqrt{2}$ is the dipole matrix element, with a_0 being the Bohr radius of the positronium atom. The spontaneous emission rate [33] for 2P→1S in positronium is $w = \alpha^4\left(\frac{2}{3}\right)^8 c/a_0 = 1/\tau_{2P-1S}$ =1/3.19 ns leading to a natural linewidth $\Delta\nu = w/2\pi = 50\ MHz$. The laser flux needed to make $\Omega_R\tau_{2P-1S} = \pi/2$ is $\phi = \pi\alpha^7 3^{-6}2^{-1}\hbar c^2/a_0{}^4$ = 36 W/cm^2 for positronium. Saturation of the 1S-2P transition for a single positronium atom would require only 36 μW of cw 243.021 nm light focused to a 10μm spot [34]. However, significant excitation of many Ps atoms would undoubtedly lead to power broadening of the transition and a loss of information essential to probing the BEC state. This will not be a difficulty if we simply wish to obtain evidence for the existence of BEC. The first-order thermal Doppler full width at half maximum, $\Delta\nu_{Thermal}$, of the 1S-2P transition at 10 K will be about $\Delta\nu_{Thermal}$ = 84 GHz. If the positronium density is n, the cavity thickness is L = 100 nm, and the cross-section for absorption of 243 nm light is $\sigma_{1S\to 2P} = \lambda^2/\pi = 1.88\times 10^{-10}$ cm^2, the transmission probability for light traversing the positronium cavity with frequency ν is:

$$T(\nu) = \exp\left[-n\sigma_{1S-2P}L\frac{\Delta\nu_{1S\to 2P}}{\Delta\nu_{Thermal}}(\pi\ln 2)^{1/2}\exp\left[4(\ln 2)(\nu-\nu_{1S\to 2P})^2/\left(\Delta\nu^2_{Thermal}\right)\right]\right]$$

Taking the Doppler width at 10 K to be 84 GHz, within 10 GHz of the 1S-2P resonance, the transmission is 0.19 for $n = 10^{18}$ cm^{-3} and 0.84 for $n = 10^{17}$ cm^{-3}. We will take an optical image of the sample using Lyman alpha radiation in transmission. There is sufficient contrast to easily measure the positronium density for trap densities greater 10^{17} cm^{-3}.

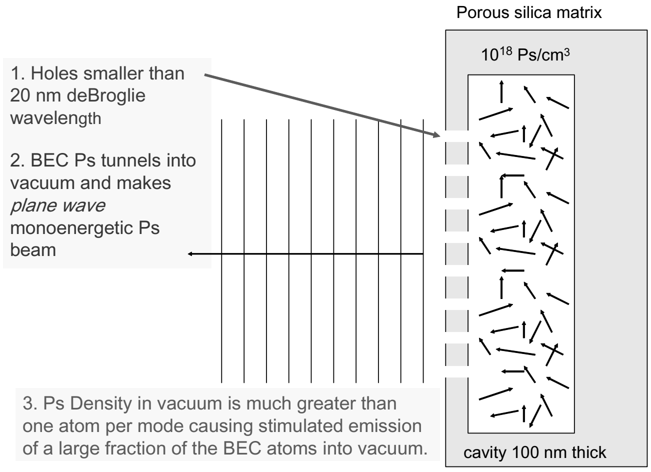

Fig. 10 A Ps atom laser may be produced via a Ps BEC.

If we have a cavity like that of Fig. 9 filled with positronium atoms at a temperature T, and there is a small hole of area l^2 in the cover then Ps atoms could tunnel out to form a coherent beam of atoms, as shown in Fig. 10. Such a hole could be fabricated using a focused ion beam. The rate of positronium leaking out of the cavity for momenta large compared to $2\hbar / l$ is approximately

$$\Gamma_{leak} = \dot{N}/N = \tfrac{1}{4}\frac{\bar{v}l^2}{dL^2} = 2.2\times10^7 \ s^{-1}\times\left[\frac{T}{10K}\right]^{1/2}\times\left[\frac{100 \ nm}{d}\right]\times\left[\frac{4\mu m}{L}\right]^2\times\left[\frac{l}{10 \ nm}\right]^2$$

where the positronium cavity has dimensions $L\times L\times d$. If $l<<d$, ground state positronium atoms effectively tunnelling [35] out through the hole in the cavity will have a velocity $v = \hbar k/2m_e = \frac{\pi}{2}\frac{\hbar}{m_e d}c = (1.82\times10^5 cm/s)\times(100\ nm/d)$ and a "tunneling" rate (by Babinet's principle in optics [36] the scattering by a hole will be the same as the scattering by a disk of the same diameter, and the effective cross section at small momentum is approximately four times the open area of the hole) estimated to be

$$\Gamma_{tunnel} \approx \tfrac{1}{4}\frac{\bar{v}4l^2}{dL^2} = 1.1\times10^5 \ s^{-1}\times\left[\frac{100 \ nm}{d}\right]\times\left[\frac{4\mu m}{L}\right]^2\times\left[\frac{l}{10 \ nm}\right]^2 .$$

The ratio of the two rates implies that about 1% of the ground state atoms would be emitted if 2/3 of the positronium is in the BEC state at 10 K. Of course the actual emission rate will increase exponentially with time due to stimulated emission of the positronium into the few available modes in the vacuum given the narrow energy spread of the Ps atoms. The narrow vacuum positronium velocity distribution could be measured with 1S-2P excitation and 2P ionization detected with great sensitivity via a channel electron multiplier sensitive to the liberated positrons.

Once a Ps atom laser has been obtained it might be possible to use it to make an antimatter gravity interferometer. A Mach-Zender (see Fig. 11) atom interferometer would be ideal for measuring small phase shifts caused by fields applied unequally to its two paths. The phase shift difference caused by the acceleration of gravity [37] for an atom traversing two paths in a vertical plane is the area enclosed by the paths times $mg/\hbar v$, where m is the particle mass, g is the acceleration of gravity and v is the particle's velocity, assumed to be changing only slightly along the paths. For positronium atoms emanating from a cavity of small dimension d, the positronium velocity in vacuum will be $\pi\hbar/d$ and the phase shift in going around the interferometer is $\Delta\phi = Amg/\hbar v$. Thus an interferometer with about 20 cm legs would be enough to see if the gravitational response of positronium is the same as that of ordinary matter. The positronium would have to be excited into the n=25 state with orbital angular momentum equal to $24\hbar$, so that its lifetime would be about 1 ms [38].

An interferometer would also be useful for determining the charge of the positronium atom via the Bohm-Aharanov effect. The fringe shifts caused by a 10 T solenoid passing through the interferometer would be sensitive to a charge imbalance of as little as one part in 10^{15}, much better than implied by the equality of the electron and positron g-factors to 6×10^{-11}, but not so sensitive to a possible CPT-violation as the $K^0\overline{K^0}$ mass difference.

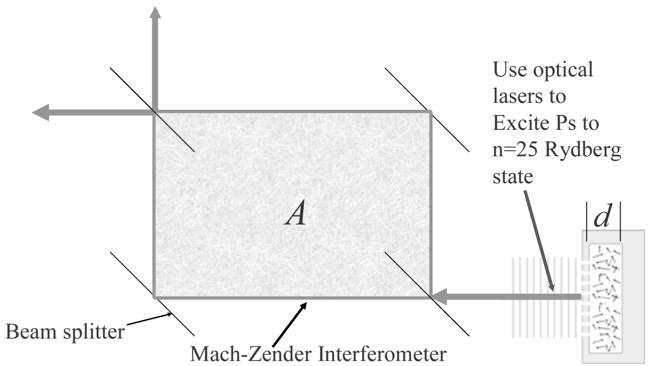

Fig. 11 A Ps atom beam laser could be used in a Mach-Zender interferometer to measure the Ps gravitational interaction.

5 Conclusion There are several other interesting experiments to be done with high density positronium, including searching for stimulated emission (requiring perhaps 10^{11} positrons in one pulse) and perhaps making an annihilation gamma ray laser using a dense Ps BEC [39]. We are currently scaling up our positron source to make such experiments more feasible.

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[34] It is interesting to note that positronium could be heated by relatively intense ns Lyman-alpha pulses to assist its collection into the cavity if necessary. About 10 nJ would be sufficient to heat 10^7 positronium atoms to room temperature without heating the porous structure significantly.
[35] Note that while the Ps is not excluded by a repulsive potential from going through the small hole, its wave function exhibits positive curvature in the neighborhood of the hole just as it would in the presence of a tunnel barrier.
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