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Prospects for making a Bose-Einstein-condensed Positronium Annihilation Gamma Ray Laser

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Abstract. An annihilation gamma ray laser could be made by a cylinder of high density cold singlet Ps annihilating into a *coherent* gamma ray burst directed along the axis of the cylinder. Such a laser would have many important uses and prospects seem fair for making a 1J model in the immediate future. Higher intensity lasers that would be useful for controled fusion are envisioned, but involve so many orders of magnitude increase in our ability to produce and control antimatter that no reasonable statement about the possibilities can be made at this time. This paper describes our vision and we briefly report the present status of the experiments.

I. Introduction.

The ultimate aim of the work contemplated in the title is to design and perfect a powerful laser based on the coherent annihilation [1-3] of a Bose-Einstein condensate of positronium [4], the hydrogen-like atoms formed from bound electron-positron pairs. The ultimate goal would require many advances that are far beyond the present state of the art. However, with almost-existing techniques and a few years of sustained effort we should be able to observe stimulated annihilation radiation, the precursor to laser emission.

Work toward obtaining and manipulating significant quantities of antimatter began many years ago with the development of positron storage devices by Surko and coworkers [5]. Reasonable extrapolations suggest that one might be able to make a positron storage device for handling up to 10^{12} positrons. We have configured a Surko trap and solid neon-moderated 35 mCi Na^{22} source so that it produces 3 nsec bursts of 8×10^7 positrons that may be focused to a 1 mm diameter (FWHM) spot in a 400 G field. With a 100 mCi source and one stage of transmission brightness enhancement [6] we should be able to form a gas of 10^7 spin-polarized triplet positronium atoms within a $10\times10\times 0.1\ \mu m^3$ cavity. The positronium gas should form a Bose-Einstein condensate for cavity temperatures below about 10K, since the BEC critical temperature for non-interacting particles of density n and mass m equal to twice the mass of an electron is given by $kT_c = [ng_{\frac{3}{2}}(1)]^{\frac{2}{3}} 2\pi\hbar^2/m \approx \left[n/10^{18}cm^{-3}\right]^{\frac{2}{3}} \times 14.62K$.

To obtain the necessary conditions for observing the stimulated emission of annihilation gamma rays will require combining a number of new or improved processes into one experiment: (1) A plasma of at least 10^{12} polarized positrons must be assembled in a Surko trap and compressed via some combination of plasma rotation, brightness enhancement and bunching; (2) The positrons must be deposited in less than one triplet Ps lifetime (peak current = 100 mA) into a cavity 1mm long and about 1 micrometer in diameter; (3) The implantation energy of the positrons must be dissipated, either by balistic phonon emission or possibly by using a sacrificial layer of material; (4) The minority positron spin component must be removed by annihilation if we want a simple and long-lived one-component condensate, possibly through Ps_2 formation at the surfaces of the positron cavity; and (5) To induce the laser pulse, the spins of the remaining polarized triplet Ps must be flipped to the singlet state in about 0.1 nsec using an intense 200 GHz microwave pulse or a 0.1 nsec 30 T magnetic field pulse. Finally, we must observe the angular distribution of the annihilation photons in the direction of the cavity axis. One way to accomplish this is via an x-ray image intensifier tube with a 0.4 mm CsI photocathode and CCD camera readout.

A gamma ray laser would have several applications in fields ranging from metrology to munitions, and some of these will be discussed in the next section. We will review some basics of a super radiant Ps laser in Section III and discuss some of the physical constraints on the structure of an annihilation laser in Section IV. We say a few words about Bose-Einstein condensation in section V, present some results on positron trapping in section VI and our conclusions in section VII.

II. Utility of annihilation photon lasers.

An annihilation gamma ray laser would produce a coherent gamma ray pulse with a narrow angular spread. A gamma ray laser might be useful for destroying distant objects such as missiles and space debris, provided the laser could deliver an energy on the order of 1GJ/m^2 to a target at a distance of 1000 km. It could also be useful for initiation of a fusion reaction for electric power production if frequent pulses of 100 MJ were available (Fig. 1). While we may never have the resources necessary for such powerful lasers, operation at a much lower energy level ought to be feasible. A 1 J pulse gamma ray laser would need about 10^{14} positrons for each pulse and would have several uses, some of which are outlined as follows:

Observe photon-photon scattering in vacuum with counter-propagating 1J pulses. The "box diagram" yields an extremely small cross section for the scattering of visible light photons. However, close to the singlet Ps resonance, the cross section approaches the unitarity limit, about 10^{-20} cm^2. Two counterpropagating 1 J pulses, 10 µm in diameter, would mutually scatter an easily observable 10% of their photons into all directions.

Measure the Compton wavelength of the electron $\lambda_C = h/mc$ and the fine structure constant a with amazing precision. The natural linewidth of an annihilation laser would be

$$\Delta\lambda/\lambda = \Delta E_{FWHM}/E = \tfrac{1}{2}\alpha^5 = 1.035\times 10^{-11} \tag{1}$$

The unperturbed wavelength of an annihilation laser is

$$\lambda_A = [1 + \tfrac{1}{4}\alpha^2 + O(\alpha^4) + \cdots]\lambda_C, \tag{2}$$

where the correction to λ_A due to the Ps binding energy is 10 ppm and the higher order corrections are from relativistic level shifts of the singlet positronium binding energy, the finite size of the Ps confinement cavity, and interactions of the singlet positronium atoms. A combined measurement of the infinite mass Rydberg, R_∞ and λ_A could yield a value for the product, $R_\infty \lambda_C/2\pi = \alpha^{-4}$ *with precision improved by four orders of magnitude over current levels*. One could make a direct measurement of the annihilation photon wavelength using a Mach-Zender interferometer fashioned from a single crystal of Ge, in which one of the arms is displaced a distance while fringes are being counted, rather like Deslattes and Henins [7], except the crystal displacement would be for determining the annihilation photon wavelength directly, not for measuring the periodicity of the diffracting crystal.

Remote detonation of primers could be accomplished by a 1 J laser pulse having a 10 µm cross section after passing through several cm of steel.

III. Basics of positronium super radiant lasers. Any laser has its roots in the phenomenon of stimulated emission, which in turn is based on the fact that the partial rate for emission of a particle obeying Bose statistics into a state that already has N particles is $N+1$ times the rate for $N = 0$, $\gamma_N = (N + 1)\times\gamma_0$. This result was first obtained by Einstein and is a requirement of time-reversal invariance of the emission and absorption processes. The partial rate may be written in terms of a cross section σ_s for stimulated emission, $\gamma_N = N\sigma_s v/\Omega + \gamma_0$, where Ω is the volume containing the N bosons and v is the emitted particle velocity. If τ is the lifetime of the decaying system, the volume is roughly $\Omega \approx (c\tau)^3$. The total number of modes $\mathscr{M}$ is roughly the number of circles of radius λ that could be drawn upon the surface of a sphere of radius $c\tau$, $\mathscr{M} \approx 2\pi(c\tau/\lambda)^2 \approx 10^{21}$, where the factor 2π was chosen so that this rather hand-waving derivation will give the exact answer for the stimulated emission cross section. We now write for the partial decay rate per mode, $\gamma_0 = (\tau \mathscr{M})^{-1} = \lambda^2/(2\pi c^2 \tau^3)$. Since v=c, we thus have *for Ps at rest and photons exactly on resonance*:

$$\sigma_s = \Omega\gamma_0/c = \lambda^2/2\pi = 2\pi\alpha^2 a_0 = 0.937\times10^{-20}\text{cm}^2. \text{ (where } a_0 \text{ is the Böhr radius)} \qquad (3)$$

This cross section is at the unitarity limit, i.e. σ_S is as big as it can possibly be!

Obviously an annihilation laser has to be made without the benefit of the usual mirrors. The idea of a mirrorless laser has its beginnings with the concept of Dicke super-radiance [8], and a suitably shaped super-radiant array of emitters can become a traveling wave laser. A photon traveling through a gas of Ps at rest with density n gathers more photons into its mode because of stimulated annihilation. Initially, the number M of photons in the laser mode grows exponentially with x, the distance traveled: $d \ln M / dx = n\sigma_S$. When an amplified photon pulse becomes large, M stops growing exponentially. All the Ps atoms in the path of the laser beam annihilate into the laser mode and all the available energy is swept into the laser beam. Gain saturation occurs when $M > \pi a^2/\sigma_S$ so that the saturation intensity is [9]

$$I_{sat} = M\, mc^2\gamma_{ann}/\pi a^2 = mc^2\gamma_{ann}/\sigma_S = 7\times10^{16}\ \text{Wcm}^{-2}. \qquad (4)$$

This does not mean that intensities higher than I_{sat} are prohibited, but simply that exponential growth ceases and all remaining Ps atoms annihilate into the laser mode when the intensity is greater than I_{sat}.

The Ps has to have very low velocities so that the Doppler shift of the annihilation photons is less than the line width $v/c < \Delta E/E = ½\alpha^5$. The only possibility is for the Ps to be in the ground state of its container, i.e. in the Bose-Einstein condensed state, as pointed out by Liang and Dermer [3]. Compton scattering of 2γ annihilation photons with stationary electrons or positrons has a cross section [10] $\sigma_C = 4\pi r_0^2[\frac{10}{9} - \frac{3}{4}\ln 3] = 0.29\times10^{-24} cm^2 \approx 3\times10^{-5}\sigma_S$, which means that one could relax the Doppler shift requirement by a factor of $\sim10^4$ to $v/c < 10^{-7}$ once the intensity exceeds I_{sat} by at least a factor of 10^4.

Antecedants. The annihilation laser had its beginings in 1930 with the work of Dirac [1] in which he used a calculation of the stimulated annihilation rate to calculate the electron-positron annihilation cross section. Almost half a cnetury later, Varma [2] suggested that there might be a degenerate e^+e^- plasma column laser in the Crab pulsar. Subsequently, Bertolotti and Sibilia [11] suggested a traveling wave geometry because of the "problem of mirrors." Loeb and Eliezer [12] and Liang and Dermer [3] suggested an annihilation laser based on laser-cooled BEC Ps. Finally, Platzman [13] and Platzman and Mills [4] pointed out that dense Ps in a cavity will form a BEC without the need for laser cooling.

Problems. There are many factors that could hinder an attempt to make a gamma ray laser. These include, heating of the BEC Ps gas due to annihilation and the possible instability of the BEC to breakup into incoherent pieces. Likewise, density fluctuations leading to refractive index variations could make the annihilation gamma ray laser unstable. There may be a heating effect associated with the microwave or magnetic field pulse. Furthermore, the production, trapping, storage, spatial compression, and filling a cavity with more than 10^{15} positrons is not solved, nor is the problem of the melting of a positron target upon implantation of so many positrons.

IV. Physical requirements for a BEC laser.

The annihilation laser we contemplate has the geometry of a cylinder of length d and radius a containing N singlet Ps atoms in the ground state of the container. (The Ps number density is $n=N/\pi a^2 d$.) Laser operation depends on the values of these three parameters, and we seek conditions that will demonstrate stimulated emission and significant gain for the smallest values of N. Various factors determining the choice of parameters are discussed below.

Doppler broadening due to the uncertainty in momentum of the Ps BEC confined in a cavity reduces the stimulated cross section compared to the ideal value of Eq 3. The axial component of the Ps ground state wave function will be proportional to $\sin(\pi z/d)$ neglecting the repulsive Ps-Ps interactions. The full width at half maximum of the square of the momentum space wavefunction is $\Delta k \approx 2\pi/d$ which means that the Doppler width is $\Delta v/c = \lambda/2d$. The transverse Doppler width is much greater than the longitudinal width, but it is reduced in importance by the factor a/d since only forward-directed photons are being significantly amplified. The transverse Doppler width is then approximately equal to the longitudinal width and increases the total Doppler width by a factor of $\sqrt{2}$, if we approximate the Doppler profiles by Gaussians. Assuming the total Doppler width is greater than the natural line width, which will be true for $d < 33$ cm, the number of photons in the laser mode grows with distance x as

$$d \ln M / dx = n\sigma_S d\alpha^5 / (\sqrt{2}\lambda_C) . \tag{5}$$

Bee's eye condition. To get approximately single-mode laser emission for a cylindrical laser medium of radius a and length d, the geometrical angular resolution, $\Delta\theta_G = a/d$ must equal the angular spread due to diffraction, $\Delta\theta_D = 1.22\lambda_C/a$. Thus the optimal tube radius is the geometric mean of the tube length and the Compton wavelength of the electron [14]:

$$a = [1.22d\,\lambda_C]^{1/2} = 172 \text{ nm x } [d/1 \text{ cm}]^{1/2} . \tag{6}$$

Initiating photon condition. In order that at least one annihilation photon is traveling along the tube of positronium so that the amplification process will start, we need the total number of BEC Ps atoms, N, times the solid angle presented by the Ps confinement cylinder to be greater than one. Combining this with the bee's eye condition, we then have the requirement

$$N > d^2/a^2 = 3.3d/\lambda_C = 1.25\times10^9 \times [d/1\ cm] \tag{7}$$

Gain. From Eq. 5 and 6, the log of the gain g will be $\ln\{g\} = N/N_0$, where N_0 is independent of d:

$$N_0 = 2.44\sqrt{2}\pi^2\alpha^{-5} = 1.64 \times 10^{12} \tag{8}$$

Thus the initiating photon requirement will be satisfied when there is significant gain.

Spin exchange rate. The condition on the singlet Ps density n such that interaction effects due to hard core repulsion (scattering length $\delta \approx 2a_0$) do not significantly reduce the fraction of the atoms in the zero-momentum state is $n < \delta^{-3} \approx 10^{24}$ cm^{-3}. At much lower densities, the destruction of singlet Ps due to spin exchange collisions will be more important. The singlet Ps atoms at density n will have a repulsive energy that is roughly

$$U = \hbar^2 n^{2/3} / 4m = m\bar{v}^2 . \tag{9}$$

If U is less than the Ps hyperfine interval, spin exchange cannot occur, which will be true for $n < 2.6\times10^{18}$ cm^{-3}. The cross section for spin exchange will be roughly $\sigma_X = \pi a_B^2$. The singlet Ps loss rate due to spin exchange collisions in units of the singlet annihilation rate will then be

$$f = 2\hbar n\sigma_X \bar{v} / mc^2\alpha^5 = \pi n^{4/3} a_B^4 \alpha^{-3} . \tag{10}$$

The condition on the density such that $f < 1$ is

$$n < n_0 \equiv \pi^{-1} a_B^{-3} \alpha^{9/4} = 4.5 \times 10^{19} cm^{-3} . \tag{11}$$

At the density n_0, T_C would be 185 K. Eq 6 then implies the length d must be greater than

$$d > [N / 1.22\pi\lambda_C n_0]^{1/2} \approx (N/N_0)^{1/2} \times 4.1\,cm . \tag{12}$$

From Eq 6, the Ps cavity would have a diameter of about 0.63 μm×$(N/N_0)^{1/4}$.

Minimal configuration for observing stimulated emission. The number of photons Q detected in the stimulated emission cone of solid angle $a^2/4d^2$ is

$$Q = \varepsilon \exp\{N/N_0\} Na^2/2d^2 \approx \varepsilon [1 + N/N_0][Na^2/2d^2] \tag{13}$$

where ε is the detection efficiency. In order that the number of detected stimulated emission photons be observable as a 1 standard deviation effect compared to the number of photons detected in a random direction we require

$$N > [N_0 2^{1/2} d/\varepsilon^{1/2} a]^{2/3} = [N_0^2 2d/1.22\varepsilon\lambda_C]^{1/3}. \tag{14}$$

Taking the equality for d in Eq 12, we then have

$$N > 2^{-1/2} 1.22^{-4/5} \pi^{-1} \varepsilon^{-2/5} \alpha^{-1/20} N_0 = 3.15 \times 10^{11} \varepsilon^{-2/5}. \tag{15}$$

With $\varepsilon \approx 1$, $N/N_0 \approx 0.2$ and $d \approx 1.8$ cm. It thus appears that we should be able to observe stimulated emission given 3×10^{11} p-Ps atoms confined to a laser cavity 2 cm in length and about one-half micron in diameter. The detector would be a single 10x10 array of 1x1 mm^2 BGO crystals 1 inch long coupled to a CCD camera and located at a distance of 25 m from the Ps cavity.

VI. Experiments

We have acquired a Surko trap and have begun experiments on high density positronium. Fig. 1 shows the results of our accumulation and compression experiments following the work of Greaves and Surko [5]. Fig 1a displays the cross section of the accumulated positron plasma as it is compressed by increasing rf levels. Fig 1c shows that the number of accumulated positrons is limited by the depth of the potential well and that more positrons can be stored if the well is increased during accumulation. A rough estimate suggests that we have achieved about 3% of the Brillouin limiting density in a 400 G field.

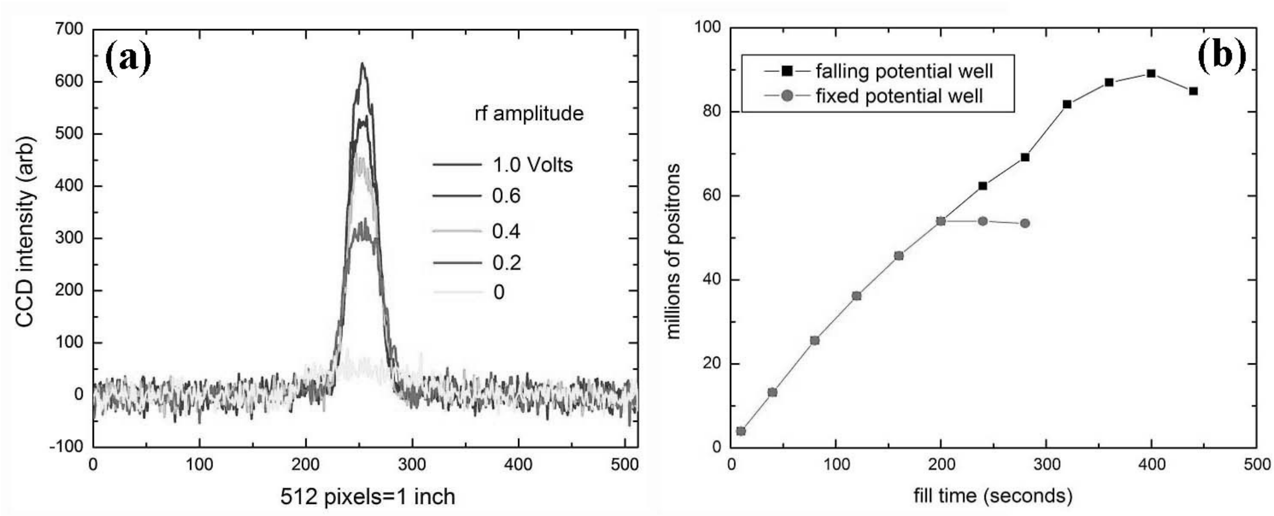

Figure 1. Positron accumulation and compression experiments. (a) Compressed pulse shape vs. rf amplitude at a frequency of 4 MHz. (b) Central density *vs.* fill time, showing the increase associated with allowing the accumulator potential well to fall as more positrons are gathered. The positron lifetime in the accumulator with 225 mV rf at 3 MHz is 265±14 sec. The accumulator magnetic field is 400G. The positron source is 35 mCi Na-22 moderated by solid Ne

VII. Conclusion

The conclusion of all this is that a BEC annihilation laser might be feasible and would be very useful and interesting to make. Our rough estimates for when some of this work could be done, given sufficient support, are: 1 year to make Ps_2 molecules, 2 more years to make a Ps BEC, and 5 more years to produce some evidence for stimulated emission of annihilation radiation and perhaps a few mJ laser. It is very exciting to think that even after the 50th anniversary of Deutsch's discovery of the lightest element, positronium [15], there are yet amazing things to be done with our favorite atom.

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