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Positronium molecule formation, Bose–Einstein condensation and stimulated annihilation

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Abstract

Low energy positron production and storage may be used to make and observe some interesting many-positron, many-electron systems both in vacuum and in the presence of ordinary matter. One of the most interesting possibilities is to make an annihilation photon laser. One might start on the path to this laser by creating a high density positron burst and forming a dense gas of positronium atoms within a cavity in a solid. This will pave the way for subsequently demonstrating the near room temperature Bose–Einstein condensation of a dense gas of positronium and establishing conditions under which stimulated emission of annihilation photons can be observed.

1. Introduction

The amazing properties of our world are the result of many-particle interactions that occur at the usual densities of condensed matter, gases and plasmas. Although nearly identical properties would be found in a mirror world of antimatter, a mixture of ordinary matter and antimatter at normal densities would be expected to display some effects that are qualitatively similar to those exhibited by ordinary matter, but under unusual conditions and with extraordinary properties. In addition, we might expect new behavior associated with the annihilation force and lack of exchange forces between particles and antiparticles. Positrons are the most easily obtained type of antiparticles, and thus an electron–positron system is the most suitable for studying a mixture of ordinary matter and antimatter. At this time we have the technology, based on low energy positron production and storage, to make and observe some interesting many-positron, many-electron systems both in vacuum and in the presence of ordinary matter. One of the most interesting suggestions is the possibility [1–4] of making an annihilation photon laser. A first step on the path to this laser would be to create a high density positron burst, as evidenced by the production of positronium molecules, and form a dense gas of positronium atoms within a cavity in a solid. This will pave the way for subsequently demonstrating the near room temperature Bose–Einstein condensation (BEC) of a dense gas of positronium and establishing conditions under which stimulated emission of annihilation photons can be observed.

The steps leading eventually to the annihilation photon laser, would pass the following milestones:

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1. Compress a cold, spin-aligned, non-neutral plasma of positrons trapped in a magnetic field to the Brillouin limit. Extract the positrons and focus them to a microscopic spot on a sample in a short time pulse.
2. Demonstrate the existence of a high positron surface density by forming and observing the dipositronium molecule, Ps_2.
3. Form a gas of high-density triplet Ps within a cavity.
4. Measure the temperature of dense Ps in a cavity.
5. Cool the high density Ps and observe its BEC.
6. Convert the Ps suddenly to the singlet state and observe stimulated emission of the annihilation photons.
7. Increase the Ps density and number to get significant annihilation photon gain and scale up to a high power annihilation photon laser.

The final goal, an intense pulsed source of coherent annihilation photons, would have applications in the fields of X-ray holography and material modification. An intense burst of annihilation photons would enable us to achieve extremely high temperatures in a short time, would be useful in the maintenance of nuclear weapons and possibly as a substitute for the fission source ordinarily used to start a fusion reaction. This could be the basis of a clean bomb or more preferably a fusion power plant. Unlike visible photons, annihilation photons have a several cm absorption length and would impart their energy to a relatively large mass when impinging on a solid target. The obvious military application to essentially instantly destroying something far away could be limited to objects high above the earth's surface because of the range limitation caused by Compton scattering in air, the mean free path of a 511 keV photon being about 80 m in air at sea level. However, the range might be extended at sea level in clear conditions by a UV laser pulse that would make a low density path for the gamma rays. In addition, a high-power pulse might induce transparency because its leading edge would tend to scatter all the electrons out of the path of the remaining photons.

2. Detailed considerations

In the following sections, I outline some thoughts on how one might reach the first six milestones.

2.1. Focusing of a uniform plasma column in a magnetic field

The necessary apparatus would employ a Surko-Greaves positron trap and a positron buncher. The compression of a cold positron plasma by a rotating electric field has recently been achieved by Greaves and Surko [5]. The plasma radius of a collection of 10^7 positrons in a 900 G magnetic induction was reduced from 3.3 to 0.7 mm by a rotating dipolar electric field. It is possible that one could come closer to the 32 μm Brillouin limit for the plasma radius by a different choice for the configuration of the rotating electric field. If the Brillouin limit is achieved, the positrons can be extracted from the magnetic field and brought to a focus, the diameter of which is only limited by the plasma temperature. These statements are supported by the following brief discussion [6].

2.1.1. Details on focusing of a uniform plasma column

We consider a tube of uniform density charged particles in vacuum at zero temperature rotating at an angular frequency ω in a magnetic induction $\boldsymbol{B}$. The particle density is n, the particle charge is the elementary charge e, the mass is the electron mass, m_e, the radius of the plasma column is a, and its length is l. The plasma is contained at the center of a long conducting tube of radius b. The total number of particles is $N = n\pi a^2 l$.

Neglecting end effects, the radial electric field is

$$E = 2\pi ner\left[\Theta(a-r) + \Theta(r-a)\frac{a^2}{r^2}\right], \tag{1}$$

where $\Theta(x)$ is the Heaviside unit step function. At equilibrium, the force on an individual particle then satisfies $e(v/c)B = eE + mr\omega^2$ which implies from Eq. (1) that

$$\omega = \frac{1}{2}\omega_L\left\{1 - \left[1 - \frac{n}{n_{BL}}\right]^{1/2}\right\}, \tag{2}$$

where the cyclotron frequency is $\omega_L = eB/m_e c = 2\pi \times [2.79935\ \text{GHz}] \times [B/1\ \text{kG}]$ and n_{BL} is the Brillouin density limit where the rest energy density of the particles, $nm_e c^2$, equals $B^2/8\pi$, the magnetic field energy density: $n_{BL} = B^2/8\pi m_e c^2 = 2.79935 \times 10^{10}\ \text{cm}^{-3} \times [B/1\ \text{kG}]^2$. Note that at the Brillouin limit, the rotation frequency of the plasma tube is half the cyclotron frequency of a single particle.

The Lagrangian L for the breathing mode motion of the charge tube is the sum of:

(1) the electrostatic energy,

$$U_E = \int \frac{E^2}{8\pi} d^3 r = \frac{N^2 e^2}{l}\left[\frac{1}{4} + \ln\left(\frac{b}{a}\right)\right];$$

(2) the magnetic energy less the energy of the external induction B,

$$U_B = \int \frac{[2BB_{\text{plasma}} + B^2_{\text{plasma}}]}{8\pi} d^3 r = -\frac{1}{4} Nma^2 \omega_L \omega + \frac{1}{48} Nma^2 (\omega_L \omega)^2 \left(\frac{a_{BL}}{c}\right)^2,$$

where we note that the second term is the magnetic self energy of the plasma and is negligible unless the edge of the plasma is becoming relativistic

$$\int \frac{B^2_{\text{plasma}}}{8\pi} d^3 r \approx \left[\frac{B}{10^8 T}\right]^2 \left[\frac{a_{BL}}{a_B}\right]^2 U_B,$$

where

$$a_{BL} = \left[\frac{8mc^2 N}{lB^2}\right]^{1/2} = 1\ \text{mm} \times \left[\frac{N}{1.759 \times 10^{10}}\right]^{1/2} \times \left[\frac{1\ \text{kG}}{B}\right] \times \left[\frac{20\ \text{cm}}{l}\right]^{1/2}$$

is the radius of the plasma at the Brillouin limit and a_B is the Bohr radius of the hydrogen atom;

(3) the azimuthal kinetic energy,

$$U_\phi = \tfrac{1}{4} Nma^2 \omega^2;$$

and

(4) the radial kinetic energy,

$$U_R = \frac{1}{4} Nm \frac{\partial a}{\partial t}.$$

The sum of all these contributions is

$$L = T - V = U_R + U_\phi + U_E + U_B = \frac{1}{4} Nm\mathring{a}^2 + \frac{1}{4} Nma^2 \omega^2 + \frac{N^2 e^2}{l}\left[\frac{1}{4} + \ln\left(\frac{b}{a}\right)\right] - \frac{1}{4} Nma^2 \omega_L \omega, \tag{3}$$

where $\mathring{a} \equiv \partial a/\partial t$. The canonical angular momentum,

$$\frac{\partial L}{\partial \omega} = \frac{1}{2} Nma^2 \left[\omega - \frac{1}{2}\omega_L\right] \equiv \frac{1}{2} NmK, \tag{4}$$

is a constant of the motion because L has azimuthal symmetry. Notice that a plasma at the Brillouin limit has $K = 0$ and therefore zero canonical angular momentum and infinite brightness if we are neglecting the plasma temperature.

It is easy to see how the canonical angular momentum can be zero if we consider removing a plasma column from a high field region to a weak or zero field region. In passing out through the weakening field, the particles see a nonzero radial component of the magnetic induction. The Larmor force in the transition region is opposite to the direction of precession and thus if the plasma is spinning sufficiently fast in the high field region, it can have zero angular momentum in the zero field region.

Given that the radial momentum is $p_r = \partial L/\partial \mathring{a} = (1/2)Nm\mathring{a}$, the radial equation of motion $\partial p_r/\partial t = \partial L/\partial a$ may be written

$$\frac{\partial^2 a}{\partial t^2} = -\frac{1}{4}(\omega_L a_{BL})^2 \partial_a \left[\frac{1}{2}\left(\frac{a}{a_{BL}}\right)^2 + \frac{2K^2}{(\omega_L a_{BL} a)^2} - \ln\left(\frac{a}{a_{BL}}\right)\right]. \tag{5}$$

The quantity in brackets is the effective potential $V(a)$ for the plasma outer radius a. If $K = 0$, the effective potential is approximately that of a harmonic oscillator with frequency $1/2\omega_L$ and an infinite repulsive barrier at the origin. The equilibrium plasma radius is

$$a^2_{eq} = \frac{1}{2} a^2_{BL} + \frac{1}{2}\left[a^4_{BL} + \frac{K^2}{\omega^2_L}\right]^{1/2}. \tag{6}$$

A plasma column may be focused by making a change in B that is sudden compared to a Larmor period. For example, accelerate a plasma column at the Brillouin limit to a kinetic energy $T = 1/2mv^2$ and let it pass into a region of high magnetic induction B' with diameter roughly $2a_{\mathrm{BL}}$. The measure of suddenness is then given by the smallness of the parameter

$$u = \frac{a_{\mathrm{BL}}\omega_{\mathrm{L}}}{v} = \left[\frac{8Nr_0}{l}\right]^{1/2}\frac{c}{v},$$

where $r_0 = e^2/m_ec^2$ is the classical radius of the electron. For $T = 2.5$ keV, we have $v = c/10$ and $u = 0.106 \times [20\ \mathrm{cm}/l]^{1/2} \times [N/10^{10}]^{1/2}$. The attractive part of the potential in (5) increases suddenly from its equilibrium value 1/2 by the ratio $(\omega'_{\mathrm{L}}/\omega_{\mathrm{L}})^2 = (B'/B)^2$, resulting in focusing of the plasma to a theoretical radius $a_{\mathrm{focus}} = a_{\mathrm{BL}}\exp\{(B/B')^2\}$. The focus is approximately at a distance $f = \pi v/\omega_{\mathrm{L}} = \pi a_{\mathrm{BL}}/u$ from the entrance to the high field region. Fig. 1 displays the effective potential $V(a)$ for various values of the canonical angular momentum K. The theoretical exponentially tight focusing at $K = 0$ is a result of the logarithmic dependence of V on a for small values of a.

Note that in spinning a plasma column to reduce K one might be able to use a rotating quadrupole electric field, since a rotating dipole field only works if applied to part of the plasma column [5] so that the effective part of the perturbation is probably the quadrupole component due to the end effects of two opposing dipoles. Although Greaves and Surko argue that Trivelpiece-Gould modes are important, it is possible that quadrupole spin up electrodes would be useful for efficient spin-up because a collection of single mass, single sign particles has no odd multipole moments about the center of charge and mass. A dipole field only displaces the center of mass/charge of the plasma and applies no torque if we neglect higher order effects. From the above simple theoretical discussion, it appears that to spin a single component plasma to the Brillouin limit one may simply apply a rotating electric quadrupole field at a frequency below the initial rotation frequency from Eq. (3) and slowly ramp the frequency up to half the single particle cyclotron frequency.

2.2. Production and observation of positronium molecules

Positronium molecules may be formed by impinging a few ns, few keV pulse containing of order 10^9 positrons on a 100 μm spot on a clean Al(1 1 1) surface in ultrahigh vacuum. For 2.5 keV positrons penetrating about 0.1 μm, the deposited energy, ≈400 J/cm^3, will raise the local sample temperature a few hundred K, neglecting ballistic phonon transport. As discussed below, the resulting surface density of positrons will be sufficient to produce a significant number of dipositronium molecules.

When positrons are present at a high enough density near a solid surface, they will interact sig-

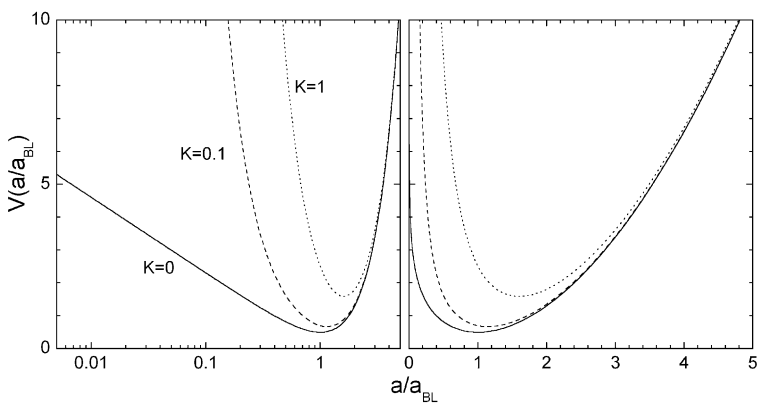

Fig. 1. Effective potential for a uniform density plasma column of outer radius a for three values of the normalized canonical angular momentum K. The effective potential is in units of $(1/4)(\omega_{\mathrm{L}}a_{\mathrm{BL}})^2$.

nificantly with each other and the solid, revealing the new physics of the many positron-many electron system. As envisioned in 1946 by Wheeler [7], the polyelectron states having n_+ positrons and n_- electrons are known to form stable bound states up to the positronium molecule Ps_2 [8]. Positronium was first observed in 1951 [9] and the positronium negative ion in 1981 [10]. As mentioned above, we are now in a position to form and study the dipositronium molecule Ps_2. The necessary experimental conditions of high positron density will also allow us to fill a small cavity with Ps atoms at a density such that BEC could occur at room temperature using our polarized positron source [11]. The positronium super fluid will be a complex system with possible distinct phases associated with the triplet and singlet positronium ground states. It seems unlikely that the fluid will condense in the absence of a container, and thus that Wheeler's vacuum polyelectron series may terminate with the Ps_2 molecule or its dimers.

2.2.1. Thermodynamics of Ps_2 formation

The binding energy of Ps_2 relative to two free Ps atoms is $\Delta E = 0.573$ eV relative to two free Ps atoms [12]. Since the activation energy for thermally desorbing Ps from its surface state on a metal is typically about 0.5 eV, Ps_2 emission will occur at temperatures below those needed for the thermal desorption of Ps atoms. The unknown factor in the formation rate of Ps_2 will be the accommodation or sticking coefficient S_{Ps_2} for Ps_2 absorption at the surface. The Ps_2 formation rate [13,14] per surface positron is proportional to the positron surface density since two positrons are involved in the reaction:

$$z_{Ps_2} = \left(\frac{\hbar m n_+}{\gamma m_+^2}\right)(S_{Ps_2})\left[\Theta\{2E_a - \Delta E\}\exp\left\{\frac{\Delta E - 2E_a}{k_B T}\right\} + \Theta\{\Delta E - 2E_a\}\right]$$
$$= \left(\frac{n_+}{n_0}\right)(S_{Ps_2})\left[\Theta\{2E_a - \Delta E\}\exp\left\{\frac{\Delta E - 2E_a}{k_B T}\right\} + \Theta\{\Delta E - 2E_a\}\right]. \quad (7)$$

Here, m_+ is the surface positron effective mass, m is the free electron or positron mass, n_+ is the surface density of positrons, S_{Ps_2} is the thermally averaged Ps_2 sticking coefficient and E_a is the activation energy for the thermal desorption of surface positrons to form Ps. The rate has been written in units of the surface positron annihilation rate $\gamma \approx 2 \times 10^9\ \mathrm{s}^{-1}$. The characteristic surface density is $n_0 = \gamma m_+^2/\hbar m = 1.73 \times 10^9\ \mathrm{cm}^{-2}$ with $m_+ = m$. The similarly normalized rate for forming Ps is [14]

$$z_{Ps} = \left(\frac{4k_B T}{\gamma h}\right) S_{Ps}\exp\left\{\frac{-E_a}{k_B T}\right\} = \left(\frac{T}{0.0235\ \mathrm{K}}\right) S_{Ps}\exp\left\{\frac{-E_a}{k_B T}\right\}. \quad (8)$$

The yields of Ps_2 molecules and thermal Ps atoms per surface positron are then

$$Y_{Ps_2} = \frac{z_{Ps_2}}{z_{Ps_2} + z_{Ps} + 1}, \quad (9)$$

$$Y_{Ps} = \frac{z_{Ps}}{z_{Ps_2} + z_{Ps} + 1}. \quad (10)$$

An Al(1 1 1) sample surface desorbs Ps with an activation energy [15,16] $E_a = 0.4$ eV and about 50% of the maximal thermal Ps yield at a temperature of about 460 K. The Ps and Ps_2 yields are calculated in Fig. 1 for various values of the surface positron density n_+. It is found that $S_{Ps} = 1$. At 500 K, assuming that $S_{Ps_2} = 1$, Fig. 2 shows that Ps_2 formation would substantially reduce the thermal Ps yield for $n_+ = 3 \times 10^{11}\ \mathrm{cm}^{-2}$. A lower

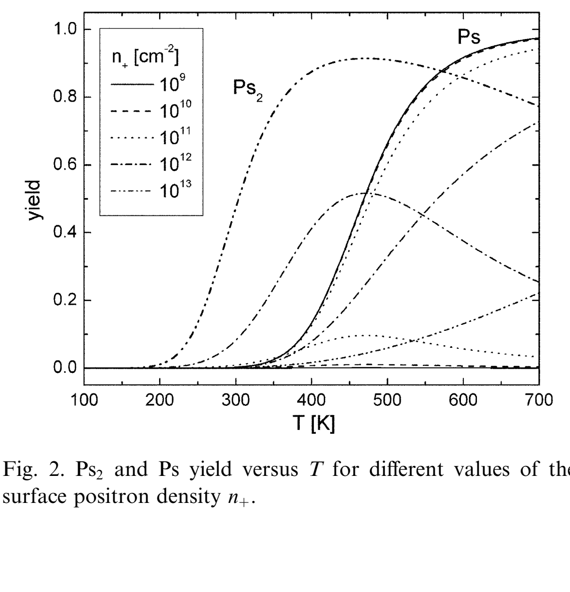

Fig. 2. Ps_2 and Ps yield versus T for different values of the surface positron density n_+.

requirement on n_+ would result from treating the Al(1 1 1) surface with oxygen [15].

2.2.2. Ps_2 detection

Ps_2 annihilates predominantly into two pairs of two photons with a rate per pair of photons very close to the spin-averaged Ps annihilation rate of 2 ns^{-1}, with a branching probability into two sets of three photons of about 0.26%. There will also be a small probability for decay into two annihilation photons plus a free electron–positron pair having kinetic energies on the order of a few eV. One may discern the occurrence of Ps_2 formation by measuring the ratio $R_{3\gamma}/R_{2\gamma}$ of three-photon to two-photon annihilations versus positron surface density, or equivalently positron focus spot size (Fig. 3). The ratio $R_{3\gamma}/R_{2\gamma}$ may be found in a single pulse many-positron event from the relative pulse heights from two scintillation detectors, one of which is shielded from the lower energy 3γ events by a few mm of Pb, and the other of which is thin and mostly sensitive to only the 3γ events. The counters should be placed as close to the positron target as possible, since one wishes to detect as many annihilation photons in one event as possible to obtain high statistical precision.

2.3. Formation of dense positronium in a cavity

It will not be a trivial exercise to obtain the high density of positronium needed for BEC to occur

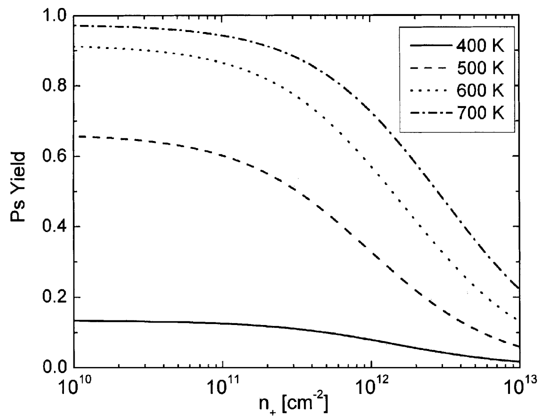

Fig. 3. Ps yield versus surface positron density n_+ at various temperatures.

by irradiating a small cavity in a positronium forming material with an intense burst of positrons. The central idea is that positrons implanted within a diffusion length of the surface of a cavity are likely to become trapped within the cavity open volume. Upon entering the cavity the positrons are likely to capture an electron from the solid to form positronium. The positronium will then thermalize due to multiple collisions with the cavity walls. If the cavity is $10 \times 0.1\ \mu m^2$ and 0.1 μm deep its volume will be $10^{-13}\ cm^3$ and we will need about 10^8 positronium atoms in the cavity to reach the nominal density needed for BEC (see Eq. (17) below). A suitable target will be a Si or quartz single crystal with an array of shallow etched cavities sealed for example by annealing it while in contact with a thin oxide-coated Si wafer. The bulk of the cover wafer is subsequently etched away, leaving an array of cavities that can be filled with positronium.

2.3.1. Difficulties associated with local heating

If a cavity is filled by impinging 3 keV positrons on the solid surface of the cavity, the energy deposited will be 5×10^{-8} J. As discussed in Ref. [17], the deposited energy will result in heating a volume, the radius of which is at most equal to the phonon mean free path [18]. However, the energy density in the volume of positron deposition, $5 \times 10^5\ J/cm^3$ will be sufficient to vaporize Si, and therefore the linear approximation which predicts that the energy will be deposited in a phonon mean free path may not be valid. It may therefore be necessary to implant the positrons at much lower energies. One possibility would to be to use a remoderator [19] that is in contact with the positronium-forming cavity surface. We coat the surface with a sacrificial layer of solid rare gas atoms thick enough to stop most of the incoming 3 keV positrons. The positrons stop, the rare gas is immediately vaporized and ionized and begins to expand away from the surface, moving about 100 μm in 10^{-7} s. An electric field of 10^5 V/cm attracts the positrons and ions towards the surface and the positrons are implanted into the surface with a mean kinetic energy comparable to the ionization potential of the rare gas atoms. The energy deposited into the cavity walls is now

100 times less and the temperature rise is only 1000 K. This scenario involves complicated plasma dynamics and needs to be carefully explored to discover conditions under which dense positronium formation in a cavity will be possible.

2.3.2. Polarized positronium

The positronium formed within a cavity is initially a collection of 3/4 triplet states, which annihilate into 3 photons with a mean lifetime of 142 ns, and 1/4 singlet states, which annihilate into two photons in 1/8 ns. If a cavity is filled with unpolarized positronium, exchange collisions amongst various pairs of positronium atoms will lead to the formation of singlet states, and thus to the rapid quenching of the triplet positronium. The positron beam of the present experiment will be partially polarized with polarization $P = (n_\uparrow - n_\downarrow)/(n_\uparrow + n_\downarrow)$, producing triplet $m = 1, -1$ and 0 and singlet Ps with populations $1/4(1+P)$, $1/4(1-P)$, 1/4 and 1/4, respectively. As argued in Ref. [11], annihilation of the singlet states and collisions amongst the various triplet substates will now cause the initially hot Ps atoms to become completely polarized into a pure $m = 1$ triplet state. In addition, if the positrons are implanted in a short burst, the formation of positronium molecules at the cavity surface will also drive up the polarization of the Ps gas by depleting the minority spin positron population. These predictions, essential for observing BEC in a gas of Ps at reasonable densities, will be tested by measuring the relative yield of three-versus-two photon events as the positron polarization is rotated by a microwave field applied transverse to the magnetic field of the positron accumulation magnet.

2.4. Measurement of the temperature of dense Ps in a cavity

To measure the Ps temperature we need to know the momentum distribution of the Ps atoms. The best way to do this would be to measure the Doppler profile for laser excitation of the atoms to the $n = 2$ state [17]. Alternately one may obtain a measurement of the temperature from the angular correlation of the 2γ annihilation photons when the gas of triplet Ps is converted into singlet Ps. The full width at half maximum of the 2γ angular correlation from singlet Ps atoms at a temperature T is

$$\Delta\theta = 2[2\ln 2]^{1/2}\left[\frac{2kT}{m_e c^2}\right]^{1/2} \approx 1\ \text{mrad} \times \left[\frac{T}{300\ \text{K}}\right]^{1/2}. \quad (11)$$

The 2γ angular correlation for Ps being converted to the singlet state in a 100 ns time interval can be measured with one or more pairs of detectors having sub ns time resolution and 1 mrad angular resolution. Each counter will register about 10 photons from a collection of 10^8 Ps, resulting in an acceptable accidental rate of about 10%.

2.4.1. Inducing triplet to singlet transitions in positronium via the ground state hyperfine transition

The $n = 1$ positronium triplet $m = 1$ state is more energetic than the $n = 1$ singlet state. The frequency splitting is known as the ground state hyperfine interval and is approximately $\Delta\nu = 203.387$ GHz. The spin Hamiltonian for positronium in a magnetic induction $\boldsymbol{B}$ is

$$H = \tfrac{1}{2} g\mu_B (\boldsymbol{\sigma}_+ - \boldsymbol{\sigma}_-)\cdot\boldsymbol{B}. \quad (12)$$

The matrix element of the x-component of the Pauli operator sum between the $n = 1$ triplet $m = 1$ state and the $n = 1$ singlet state is

$$\left\langle {}^3S_1 m = 1 \middle| (\boldsymbol{\sigma}_+ - \boldsymbol{\sigma}_-)\cdot\hat{\boldsymbol{e}}_x \middle| {}^1S_0 \right\rangle = -2^{1/2}. \quad (13)$$

The matrix element coupling these two states under the action of an rf magnetic induction $\boldsymbol{B} = \hat{\boldsymbol{e}}_x B_x \cos\omega t$ is therefore $-2^{-1/2} g\mu_B B_x \cos\omega t$.

The equations of motion for the amplitudes for being in the singlet and triplet $m = 1$ states, denoted respectively a_s and a_t in the rotating frame approximation are

$$-i\hbar\frac{\partial a_s}{\partial t} = -2^{-1/2} g\mu_B B_x \exp\{i(\omega - \omega_0)t\} a_t, \quad (14a)$$

$$-i\hbar\frac{\partial a_t}{\partial t} = \omega_0 a_t - 2^{-1/2} g\mu_B B_x \exp\{i(\omega - \omega_0)t\} a_s. \quad (14b)$$

On resonance, $\omega = \omega_0 = 2\pi\Delta\nu$, and for $a_t \approx 1$,

$$|a_s|^2 \approx \frac{1}{2}\left[\frac{g\mu_B B_x t}{\hbar}\right]^2 = 1.54693 \times \left[\frac{B_x}{1\ \mathrm{kG}}\right]^2 \times \left[\frac{t}{10^{-10}\ \mathrm{s}}\right]^2. \quad (15)$$

Thus the rf field that would yield a full transition from triplet to singlet in a time slightly less than the singlet mean lifetime of 1/8 ns corresponds to an intensity $I = B_x^2 c/8\pi \approx 1.2 \times 10^8$ W/cm^2. Focused to an area $\lambda^2 = 2$ mm^2 the rf pulse that would make a triplet to singlet transition would have a power of 3 MW and a minimum pulse energy of 300 μJ. The power and pulse energy requirements can be reduced by an order of magnitude by using a build-up cavity with $Q = 10$. According to Wilson [20], 25 kW 5 ns pulses at 200 GHz are available. These pulses have sufficient energy but are a little lower in intensity than needed for inducing stimulated emission from a Bose–Einstein condensate.

An rf field of about 30 times less power (~30 kW) would yield a full transition from triplet to singlet in one triplet mean lifetime of 1/7 μs.

When a Ps atom makes a transition from the triplet to the singlet state it emits a 0.8 meV photon and therefore recoils. The 0.8 meV/c recoil momentum is negligible compared to the momentum distribution associated with thermal motion and thus will not affect the measurement of the Ps temperature. In fact, if the Ps is in a cavity with dimensions small compared to the 1.5 mm wavelength of the 0.8 meV photon, the momentum will be transferred to the cavity walls rather than to the Ps center of mass motion.

2.5. Observation of Ps Bose–Einstein condensation

A gas of triplet $m = 1$ positronium of density n in a cavity will have a macroscopic fraction f of its atoms in the ground state of the cavity for temperatures T less than a critical temperature [11,17]

$$T_c \approx \frac{1}{2}\frac{\pi\hbar^2 n^{2/3}}{m_e k_B} = 734\ \mathrm{K} \times \left(n 10^{-21}\ \mathrm{cm}^3\right)^{2/3}, \quad (16)$$

$$f = 1 - \left(\frac{T}{T_c}\right)^{3/2}. \quad (17)$$

Having made a dense Ps gas in a cavity, we may look for its BEC by measuring the momentum distribution of the Ps atoms, by slowly converting the triplet Ps atoms into singlet atoms as in Section 2.4. One possibility for detecting the near zero momentum of the Bose–Einstein condensed positronium would be to use a pair of X-ray image intensifiers. These are 30 cm diameter image tubes having a CsI photocathode roughly 0.4 mm thick and employ a CCD camera for readout and have ~0.25 mm gamma ray resolution. The efficiency per tube is about 1% for annihilation photons and the number of readout pixels can be as much as 2048^2. The tubes are located 2.5 m from the Ps target so that the angular resolution is about 0.1 mrad. A single event consisting of the annihilation of 10^8 singlet positronium atoms then is captured by the image intensifiers. One then counts the number of zero momentum and nonzero momentum gamma ray pairs by correlating the event patterns on the two detectors. From this one may deduce the fraction f.

There are many interesting aspects of Bose–Einstein condensed Ps. For example, there are large fluctuation corrections for a dense Ps gas near the critical temperature. As another example, an unpolarized Bose–Einstein condensed Ps gas will have a complex phase diagram because of its four spin states. The $m = \pm 1$ states are degenerate and could condense into phases made of paired Ps atoms having total spin 2 or 0. The Gerade (g) and ungerade (u) (binding and unbinding) states of the $m = 0$ Ps gas may also phase separate. An interesting question is whether the u and g states eat each other faster than they phase separate.

2.6. Experimental realization of stimulated emission of para-positronium

Liang and Dermer [4] suggested the possibility of the stimulated amplification of laser-cooled para-positronium annihilation photons. They gave the cross section per singlet Ps atom for single photon stimulated emission of annihilation photon pairs and described proposed experimental realizations. At first sight, the suggested experiment appears impractical due to the need for long (5 m to 1 km) and narrow (1 μm) columns of positro-

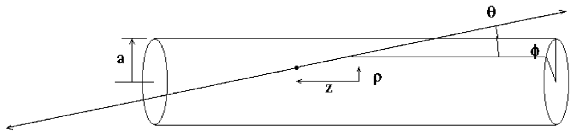

Fig. 4. Geometry of a tube of dense positronium atoms for estimating the probability of an annihilation photon passing through the full length of the column.

nium because of the low densities achievable with laser cooling and the high cross section for singlet to triplet conversion collisions. Subsequent work by Platzman and Mills [11] showed that one could achieve BEC of triplet Ps at rather high densities simply by allowing a gas of Ps to cool to temperatures on the order of 100 K in a small cavity within a solid. The use of spin polarized positrons ensures that the singlet to triplet conversion rate is not a problem. This scheme should allow one to achieve observable gain following BEC of about 10^{12} spin polarized triplet Ps atoms in a 1 mm length cavity of radius 200 nm, as shown in Fig. 4. The annihilation photon pulse would be initiated by applying a π pulse of mm-wave radiation at the hyperfine transition frequency as suggested by Liang and Dermer.

2.6.1. Geometrical restrictions

There are several restrictions on the geometry of an experiment to observe stimulated emission.

Transverse confinement of Ps in a tube of radius a raises the lowest energy state of non-interacting Ps in the tube by $\Delta E_\perp = 2^{-4}(\hbar^2/m_e)(\pi/a)^2$. The transverse momentum $k_\perp \leqslant \pi/2a$ imparted to the walls of the cavity upon annihilation will cause photons emitted at an angle θ relative to the tube axis to be Doppler shifted by an energy $E_D = v_\perp m_e c \sin\theta$, where $v_\perp = \hbar k_\perp/2m_e$. To avoid reduction of the stimulated emission cross section, the Doppler shift must be less than the uncertainty in energy $\hbar/\tau$ caused by the 2γ annihilation rate $1/\tau = 1/2m\alpha^5c^2/\hbar$. This restricts amplified photons to have emission angles

$$\theta < \frac{2}{\pi}\alpha^4\frac{a}{a_B}. \tag{18}$$

The probability that an annihilation photon satisfies this condition is

$$P = \frac{1}{4}\theta^2 = \pi^{-2}\alpha^8\left(\frac{a}{a_B}\right)^2. \tag{19}$$

The gain experienced by a photon will on the average be

$$g = \exp\left\{\tfrac{1}{2}nd\sigma\right\}, \tag{20}$$

where n is the Ps density, d is the length of the Ps cylinder and $\sigma = 10^{-20}$ cm^2 is the cross section per singlet Ps atom for stimulated emission of an annihilation photon. The total number of atoms is N and the total number of photons is

$$2N = 2\pi na^2d. \tag{21}$$

For significant gain, we need a minimum Ps density

$$n > \frac{1}{d\sigma}. \tag{22}$$

In order that there be at least one amplified photon, we need a minimum total number of Ps atoms

$$N = \pi na^2d > P^{-1} = \pi^2\alpha^{-8}\left(\frac{a_B}{a}\right)^2. \tag{23}$$

Combining (22) and (23) we have

$$\pi a^2 > \frac{\sigma}{P} \tag{24}$$

or

$$a > \pi^{1/4}\alpha^{-2}\sigma^{1/4}a_B^{1/2} = 2\times10^{-5}\ \text{cm}. \tag{25}$$

The total number of Ps atoms needed is then

$$N = \pi na^2d > \frac{\pi a^2}{\sigma} > 1.2\times10^{11}. \tag{26}$$

There is a condition on the Ps density, such that interaction effects due to hard core repulsion (scattering length $\delta \approx 2a_B$) do not significantly reduce the fraction of the atoms in the zero-momentum state, namely

$$n < \delta^{-3} = 10^{24}\ \text{cm}^{-3}, \tag{27}$$

which requires that the length of the Ps column be

$$d > \frac{\delta^3}{\sigma} = 10^{-4}\ \text{cm}. \tag{28}$$

It is important to note that there is a non-zero fraction of the Ps atoms in the zero momentum state even if $n\delta^3$ approaches unity. The experimental proof of this is the case of liquid He^4 for which the atoms have a 10% probability of occupying the zero momentum state despite being at liquid densities.

The total fraction of the annihilation photons experiencing the full gain is roughly

$$P' = \left(\frac{a}{d}\right)^2, \quad (29)$$

so one should observe a pulse of

$$NP' = \frac{\pi a^4}{\sigma d} \quad (30)$$

amplified photons.

A further consideration is that the amplified gamma ray beam will be spread over an angle $\theta_D \approx \lambda/2a = h/2am_e c$ due to diffraction associated with the finite radius a of the tube containing the amplifying medium. For single mode operation, the geometry of an annihilation photon laser should be such that the geometrical and diffractive angular spreads are equal, i.e. such that

$$d = \frac{a^2}{\pi\alpha a_B}. \quad (31)$$

As an example we take $n = 10^{21}$ cm^{-3} for which $T_c = 750$ K and d needs to be 1 mm for an average gain of $e^{1/2}$ according to Eq. (22). The stimulated emission output should be the number of amplified photons from (30), about 5000, times the gain of $e^{1/2}$. Evidence for stimulated emission could be observed even with significantly fewer Ps than the $N > 1.2 \times 10^{11}$ indicated by (26). For single mode output d would need to be increased to 3.5 cm for $a = 2 \times 10^{-5}$ cm according to Eq. (31). In this case the output would be a pulse of about 10^{11} photons (about 8 mJ) with $N \approx 4 \times 10^{12}$ Ps atoms.

3. Conclusion

The final goal of scaling up to a high power annihilation photon laser does not seem impossible, but will require 10^7–10^{10} times more positrons to attain pulses in the MJ or GJ range. Our current ideas about how this is to be done might include capturing the full fast positron output of a 1 MW LINAC. The MeV positrons might be efficiently utilized by large moderator structures and elaborate switch yards for bringing the positrons to their target. Clearly, more work is needed.

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